BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

Probability and Statistical Engineering, ENEE2307

Midterm Exam

Date: 11/4/2018 Time: 90 minutes

Problem 1 (20pts) (Criterion a):

A certain form of cancer is known to be found in women over 60 with probability $\boxed{0.07}$. A blood test exists for the detection of the disease, but the test is not infallible (معصوم). In fact, it is known that 10% of the time the test gives a false negative (i.e., the test incorrectly gives a negative result) and 5% of the time the test gives a false positive (i.e., incorrectly gives a positive result).

a. For a randomly selected woman over 60, what is the probability of receiving a positive test?

b. If a woman over 60 is known to have taken the test and received a negative result, what is the probability that she has the disease?

$P(0) = \frac{1}{4}$

Problem 2 (20pts) (Criterion a):

A multiple-choice quiz contains 5 questions, each with 4 options (one is the correct answer). Assume a student just guessing (بالحزر، بخمن) on each answer.

a. What is the probability that a student will get zero in the quiz?

b. What is the probability that a student will answer less than two questions correctly?

$\left(\frac{5}{0}\right)\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5$

$1 - P$

Problem 3 (20pts) (Criterion a):

Let X be a continuous random variable that has the following cumulative distribution function

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ 0.01x^2 & 0 < x \le 10 \\ 1 & x > 10 \end{cases}$$

a. Find $P(X \le 5)$

b. Find the mean value of X.

$X = 5$

Problem 4 (20pts) (Criterion a):

Suppose that the temperature, T, measured in °C at noontime during the month of July is a normal random variable with variance of 25. Also assume that $P(T \ge 30) = \frac{1}{2}$.

a. Find the mean value of T.

b. What is the probability that T exceeds 35?

AMRO

Problem 5 (20pts) (Criterion a):

Let the random variable X have a uniform probability density function over the interval $0 \le x \le 6$. A random variable Y is defined by the transformation $Y = \sqrt{X+3}$.

a. Find the mean and variance of X.

b. Find the probability density function of Y.

Transform

Continuous → $\frac{f(x)}{\left(\frac{dy}{dx}\right)}$

جامعة بيرزيت

BIRZEIT UNIVERSITY

Electrical and Computer Engineering

ENEE2307 – spring 2018

Quiz (1)

(10)

Name: Lina   ID: 1153284

Q1. Three machines, M1, M2 and M3 produce the following proportions of a product:

Production: M1: 10%, M2: 30% and M3: 60%.

The probability the machines produce defect products is:

Defect: M1: 4%, M2: 3% and M3: 2%.

a) – If an item selected randomly, what is the probability that it is defected?

$P(D/M_1)\,P(M_1) + P(D/M_2)\,P(M_2) + P(D/M_3)\,P(M_3)$ — total Probility

$P(D) = .1 \times .04 + .3 \times .03 + .6 \times .02 =$

$.004 + .009 + ... = 0.025$ 2

b) – If the selected item is defected, what is the probability that it was made by machine M1?

$P(M_1/D) = \dfrac{P(D/M_1) \times P(M_1)}{P(D)} = \dfrac{.04 \times .1}{0.025}$ 2

$= .16$

Q2. Given that P(A)=0.9, P(B)=0.8 and P(A∩B)=0.75, find:

a) – $P(A \cup B)$ ⇒ $P(A) + P(B) - P(A \cap B)$

$.9 + .8 - .75 = .95$ 2

b) – $P(A \cap \bar{B})$ $P(A) - P(A \cap B)$

$.9 - .75 = .15$ 2

c) – $P(\bar{A} \cap \bar{B})$

$= P(\overline{A \cup B}) = 1 - .95 = .05$ 2

BIRZEIT UNIVERSITY

Faculty of Engineering and Technology

Department of Electrical and Computer Engineering

Engineering Probability and Statistics ENEE 2307

Dr. Wael Hashlamoun, Mr. Nofal Nofal, Dr. Mohammed Jubran, Dr. Abdalkarim Awad

Midterm Exam

Date: Sunday 4/12/2016
Name: [redacted]

Time: 75 minutes
Student #: 1150759

Opening Remarks:

- This is a 75-minute exam. Calculators are allowed. Books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1 (20 pts):

a. If a multiple-choice test consists of 5 questions, each with 4 possible answers of which only one is correct. Assume a student just randomly guesses (يخمن) the correct answer to each questions. What is the probability that the student gets all of them wrong?

b. A pair of coins are tossed simultaneously and independently. Each coin has a probability 0.55 to be heads (H). What is the probability that the outcomes of the two coins are different?

Problem 2 (15 pts)

In an experiment to study the relationship of hypertension (الضغط) and smoking habits, the following data are collected:

	Nonsmokers (NS)	Moderate Smokers (MS)	Heavy Smokers (HS)
Hypertension (H)	15%	19%	16%
No-hypertension (NH)	25%	15%	10%

a. What is the probability that a randomly selected person is a Nonsmoker?

b. What is the probability that a randomly selected person is both a moderate smoker and experiences hypertension?

c. If a random person is selected and found to be a heavy smoker, what is the probability that the person is experiencing hypertension?

Problem 3 (16 pts)

The waiting time, in hours, between successive speeders (المتجاوزين للسرعة) spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-8x} & x \geq 0 \end{cases}$$

a. Find the probability of waiting less than 12 minutes between successive speeders?
b. What is the average waiting time, in hours, between successive speeders?

Problem 4 (16 pts):

In testing a certain kind of truck tire, it is found that 25% of the trucks fail to complete the test run without a blowout.

a. Find the probability that out of 6 trucks tested, less than two have blowouts.
b. How many of the 6 tested trucks would you expect to have blowouts?

Problem 5 (15 pts):

Suppose that the proportion of colorblind people in a large population is 0.005. Use the normal approximation to calculate the probability that there will be at most 32 colorblind person in a randomly chosen group of 6000 people.

Problem 6 (18 pts):

Let X be a random variable representing the time (in years) it takes to develop a software. Suppose that X has the following probability density function

$$f_X(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & otherwise \end{cases}$$

$k = \frac{3}{8}$

a. Find k so that this is a valid probability density function
b. Compute the probability that it takes more than 1 year to develop the software.
c. Find the probability that it will take more than 6 months to develop the software given that it already exceeded 3 months?

Good Luck

Estimator

Chapter 9
Estimation and Application

BIRZEIT UNIVERSITY

Faculty of Engineering

Electrical Engineering Department

Probability and Statistical Engineering, ENEE2307

Dr. M. Jubran

Name: Tasneem Omain — Student #: 115239

Problem 1:

Given a random sample of size n taken from a Gaussian population with parameters; mean μ_X and variance σ_X^2. Use the Maximum Likelihood (ML) technique to derive an estimator and then try to make it unbiased for the following cases:

1. An estimator for the variance if the true mean (mean of the population) is known.
2. An estimator for the variance if the true mean (mean of the population) is unknown.
3. An estimator for the mean if the true variance (variance of the population) is known.
4. An estimator for the mean if the true variance (variance of the population) is unknown.

$$f_{X_1 X_2 \ldots X_n}(x_1, x_2, \ldots, x_n) = f_{X_1}(x_1) f_{X_2}(x_2) f_{X_3}(x_3) \ldots f_{X_n}(x_n)$$

Gaussian μ_x, σ_x^2

$x_1, x_2, \ldots, x_n$ random

$x_1, x_2 \ldots x_n$ are standard

$$f_{X_1}(x_1) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x_1-\hat{\mu}_x)^2}{2\sigma_x^2}}$$

$$f_{X_2}(x_2) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x_2-\hat{\mu}_x)^2}{2\sigma_x^2}}$$

$$f_{X_n}(x_n) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\frac{(x_n-\hat{\mu}_x)^2}{2\sigma_x^2}}$$

$$f_{X_1 \ldots X_n}(x_1 \ldots x_n) = (2\pi\sigma_x^2)^{-n/2} e^{-\frac{1}{2\sigma_x^2}\sum_{i=1}^{n}(x_i-\hat{\mu}_x)^2}$$

$$\frac{\partial f_{X_1 X_2 \ldots X_n}(x_1, x_2, \ldots x_n)}{\partial \hat{\mu}_x} = 0$$

$$\frac{\partial \ln (f_{X_1 X_2 \ldots}(x_1, x_2, \ldots x_n))}{\partial \hat{\mu}_x} = 0$$

the same $\hat{\mu}_x$

Question#1 [17 Points]

A. Two fair coins are flipped at the same time, what is the probability of getting a match (same face on both coins)?

$S = \{HH, HT, TH, TT\}$

A: Same Face on both coins

$A = \{HH, TT\}$

$P(A) = P(HH) + P(TT)$

$= 2 \times P(HH)$

$= 2 \times \frac{1}{2} \times \frac{1}{2} = \boxed{\frac{1}{2}}$

+9

B. The events A and B are defined over the sample space S. Assume A and B are statistically independent. Prove that $\overline{A}$ and $\overline{B}$ are also statistically independent. Hint: Show that $P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$

A, B, are statistically independent

$P(A \cap B) = P(A) \times P(B)$

$P(A \cup B)^c + P(A \cup B) =$

$P(A \cup B)^c = 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(A) \dots$

+8

$(A \cup B)^c = A^c \cap B^c$

$P(A \cup B)^c = P(A^c \cap B^c)$

$P(A^c \cap B^c) = 1 - (P(A) + P(B) - P(A)P(B))$

$= 1 - P(A) - P(B) + P(A)P(B)$

$1-$

[illegible]

$P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$

$P(A \cap B) = P(A^c) + P(B^c) - P(A^c \cap B^c)$

$1 - P(A \cap B) = P(A^c) + P(B^c) - P(A^c \cap B^c)$

$1 - P(A)P(B) = P(A^c) + P(B^c) - P(A^c \cap B^c)$

$P(A^c \cap B^c) = P(A)P(B) - 1 + P(A^c) + P(B^c)$

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$(A \cup B)^c = A^c \cap B^c)$

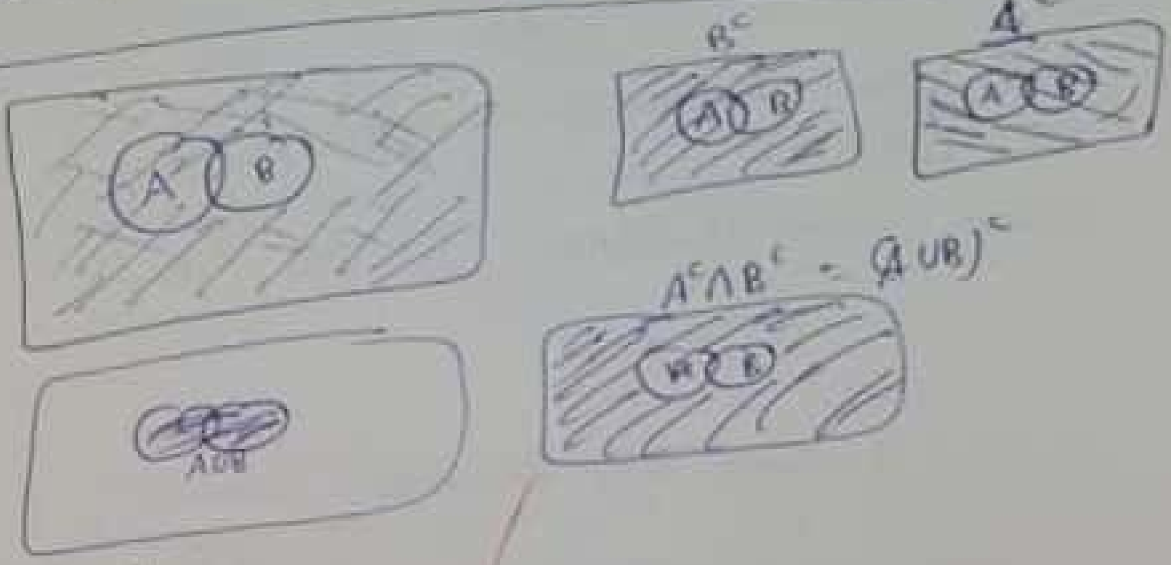
$P(A \cup B) + P(A \cup B)^c = 1$

$P(A \cup B)^c = 1 - P(A \cup B)$

$= 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - (P(A) + P(B) - P(A)P(B))$

$P(A^c \cap B^c) = 1 - (P(A) + P(B) -$



$P(S) = 1$

$1 - (1 - P(A^c)) = (1 - P(B^c)) + (1 - P(A^c))(1 - P(B^c))$

$P(A^c \cap B^c) = 1 - 1 + P(A^c) - 1 + P(B^c) + 1 - P(B^c) - P(A^c) + P(A^c)P(B^c)$

$P(A^c \cap B^c) = P(A^c)P(B^c)$

Question#2 [17 Points]

Let $f_X(x)$ be the probability density function of the random variable X.

$$f_X(x) = \begin{cases} K|x| & -1 \le x \le 1 \\ 0 & otherwise \end{cases}$$

A. Find the value of the constant K.

$|x| = \begin{cases} x, & 0 \le x \le 1 \\ -x, & -1 \le x \le 0 \end{cases}$

$k|x| = \begin{cases} kx, & 0 \le x \le 1 \\ -kx, & -1 \le x \le 0 \end{cases}$

$f_X(x) = \begin{cases} kx, & 0 \le x \le 1 \\ -kx, & -1 \le x \le 0 \\ 0, & otherwise \end{cases}$

$\boxed{k=1}$

$\int f_X(x)\,dx = 1$ ✗ 6

$\int_{-1}^{1} f_X(x)\,dx = \int_{-1}^{0} f_X(x)\,dx + \int_{0}^{1} f_X(x)\,dx$

$= \int_{-1}^{0} -kx\,dx + \int_{0}^{1} kx\,dx = 1$

$-k\frac{x^2}{2}\Big]_{-1}^{0} + k\frac{x^2}{2}\Big]_{0}^{1} = 1$

B. Determine the mode of $f_X(x)$.

mode of $f_X(x)$ = القيمة التي تتكرر أكثر من غيرها

$\frac{d f(x)}{dx} = 0$ ✓ ×2

$\frac{d f_X(x)}{dx} = \begin{cases} 1, & 0 \le x \le 1 \\ -1, & -1 \le x \le 0 \\ 0, & otherwise \end{cases}$

$f(x) = \begin{cases} x & 0 \le x \le 1 \\ -x & -1 \le x \le 0 \\ 0, & otherwise \end{cases}$

1) while $-1 \le x \le 0$, $f(x) = -x$

$\frac{d f_X(x)}{dx} = 0$

$-1 = 0$ There is no mode in this part

No mode

2) while $0 \le x \le 1$, $f_X(x) = x$

$\frac{d f_X(x)}{dx} = 1 \ne 0$

There is no mode in this part also

C. Compute $P(X \ge 0.5)$. (leave your answer in terms of K)

$P(X \ge 0.5) = 1 - P(X = 0.5)$ ✗ 6

$= 1 - \int_{-1}^{0.5} f_X(x)\,dx$

$= 1 - \left(\int_{-1}^{0} f_X(x)\,dx + \int_{0}^{0.5} f_X(x)\,dx\right)$

$= 1 - \left(\int_{-1}^{0} -kx\,dx + \int_{0}^{0.5} kx\,dx\right)$

$= 1 - \left(-\frac{kx^2}{2}\Big]_{-1}^{0} + \frac{kx^2}{2}\Big]_{0}^{0.5}\right)$

$1 - \left(-k\left(0 - \frac{1}{2}\right) + k\left(\frac{0.5^2}{2} - 0\right)\right)$

$1 - \left(\frac{k}{2} + \frac{k}{8}\right)$

$1 - \frac{5k}{8} = \boxed{\frac{8-5k}{8}}$ $= \frac{3}{8}$

Question#3 [17 Points]

Let $F_X(x)$ be the Cumulative Distribution Function of the discrete random variable X.

$$F_X(x) = \begin{cases} G & x < -1 \\ 0.2 & -1 \le x < 2 \\ H & 2 \le x < 4 \\ R & 4 \le x \end{cases} =$$

Assuming $P(X = 2) = 0.5$, answer the following:

A. Find the value of each of the constants, G, H, and R.

$\boxed{H = 0.5}$

$\boxed{G = 0}$

$G + H + R + 0.2 = 1$

$0.7 + G + R = 1$

$G + R = 0.3$

$\boxed{R = 0.3}$

B. Determine the Probability Mass function of X.

$1 - F_X(x)$

$$= \begin{cases} 1, & x < -1 \\ 0.8, & -1 \le x < 2 \\ 0.5, & 2 \le x < 4 \\ 0.7, & 4 \le x \end{cases}$$

Question#4 [17 Points]

17

A. Birzeit University has 12,885 Bachelor students and the Faculty of Engineering and Technology at the University has 1,296 female and 1,928 male students. If two students from the Faculty of Engineering and Technology are selected randomly. What is the probability that they are female students?

$$\frac{\binom{1296}{2}\binom{1918}{0}}{\binom{3224}{2}} = \frac{\frac{(1296)!}{2!\,(1294)!} \times 1}{\frac{(3224)!}{2!\,(3222)!}} = \frac{\frac{(1296)(1295)}{2}}{\frac{(3224)(3222)}{2}}$$

$$\Rightarrow \frac{(1296)(1295)}{(3224)(3223)} \approx 0.1615$$

B. The GPA of Birzeit students is modeled as Normal Random variable with a mean of 76 and variance of 16. Birzeit University has 12,885 students, how many of them will have GPA greater than 74.

$$P(GPA > 74) = 1 - P(GPA < 74)$$

$$1 - \phi\left(\frac{74 - 76}{4}\right) = \cancel{1 - \phi(-0.125)}$$

$$= 1 - (1 - \phi(0.125)) = \phi(0.125) = 0.5478$$

No of students = 0.5478 × 12,885

= 7058.403 students will have a mark greater than 74

Q4) part b

$1 - \phi\left(\frac{74-76}{4}\right)$

$= 1 - \phi\left(\frac{-2}{4}\right)$

$1 - \phi(-0.5)$

$= \cancel{1 - \phi(1-\phi(0.5))}$

$= 1 - (1 - \phi(0.5))$

$= \phi(0.5) = 0.6915$

No of students = 8909.9775 Student ≈ 8910

Question#5 [16 Points]

16

The life time X of battery a certain electronic component is a random variable with probability density function:

$$f_X(x) = \begin{cases} e^{-x} & 0 \le x \\ 0 & otherwise \end{cases}$$

Two of these components operate independently in a device. The device operates if both components operate.

A. Find the probability that a single component operates for at least 2 hours

$P(x \geq 2) = 1 - P(x = 2)$

$= 1 - \int_0^2 f_X(x)\,.dx = 1 - \int_0^2 e^{-x}\,.dx$

$u = e^{-x}$

$du = -e^{-x}\,.dx$

$x = 2 \Rightarrow u = 0.1$

$x = 0 \Rightarrow u = 1$

$= 1 + \int_0^2 -e^{-x}\,.dx$

$= 1 + \int_1^{0.135} 1.du = 1 - \int_{0.135}^{1} 1.du = 1 - (1 - 0.135)$

$= \boxed{0.135}$

B. Find the probability that the device operates for at least 2 hours

$P(\text{device operates}) = P(C_1 \cap C_2)$

$= P(C_1) \times P(C_2)$

$= 0.135 \times 0.135 = 0.018225$

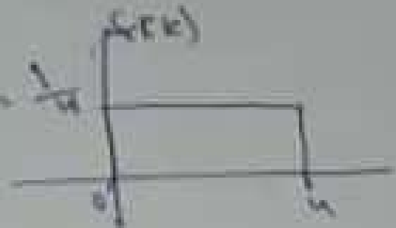
Question#6 [16 Points]

The random variable X has a uniform probability density function over the interval [0, 4]. A random variable Y is defined by the transformation $Y = (X - 2)^2$.

A. Write down the probability density function of X.

$f_X(x) = \begin{cases} \frac{1}{4} & 0 \le x \le 4 \end{cases}$ ✓ 4/4

$\frac{1}{b-a} = \frac{1}{4}$



B. Find the mean of X.

$\mu_x = \int_0^4 x\, f_X(x)\, dx = \int_0^4 \frac{x}{4}\, dx \quad \frac{1}{4}\cdot\frac{x^2}{2}\Big]_0^4 = \frac{1}{4}\left[\frac{16}{2} - 0\right]$

$= \frac{1}{4} \times 8 = \boxed{2}$

4/4

or anthor way $\mu_x = \frac{b+a}{2} = \frac{4+0}{2} = \boxed{2}$ ✓

C. Find the probability density function of Y.

$f_y(y) = \frac{f_x(x)}{\left|\frac{dy}{dx}\right|}$ ✓

14.5/16

$Y = (x-2)^2$

$\sqrt{y} = |x-2|$

$\sqrt{y} = +(x-2)$

$\sqrt{y} = -(x-2)$

$\left|\frac{dy}{dx}\right| = 2(x-2)\times 1 = 2(x-2)$ ✓

$f_y(y) = \frac{\frac{1}{4}}{2(x-2)} = \frac{1}{8(x-2)}$

$2 \le x \le 4$

$0 \le x-2 \le 2$

$0 \le (x-2)^2 \le 4$

$\boxed{0 \le y \le 4}$

$|x-2| \begin{cases} x-2, & \cancel{x \ge 2} \le x \le 4 \\ -(x-2), & 0 \le x \le 2 \end{cases}$

$x=2$

$0 \le x \le 2$

$0 \le (x-2)^2 \le 0$

$0 \le y \le 4$

6.5/8

2x

$f_y(y) = \begin{cases} \frac{1}{8\sqrt{y}}, & 0 \le y \le 4 \\ 0, & y = 0 \end{cases}$ ✓